

SELECTION RULES FOR ^{48}Cr

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Abstract

In the single j shell ($f_{7/2}$) ^{48}Cr is the first even-even nucleus for which there are $T = 0$ (isoscalar) $J = 1^+$ states and $T=1$ $J=0^+$ states. These states are studied here. This nucleus, in the same model space, is midshell for both neutrons and protons and this leads to many selection rules. A scissors mode like excitation involving both spin and orbit is identified.

1 Introduction.

If we limit ourselves to charge independent interactions, e.g. the strong interactions, we can classify nuclear ground and excited states by the isospin quantum number T . The neutron has $T = 1/2$ and projection $T_z = 1/2$ whilst the proton has $T = 1/2$, $T_z = -1/2$. A given nucleus has $T_z = (N - Z)/2$. We have the rule $T \geq T_z$, and that an isospin T corresponds to a multiplet with $(2T + 1)$ members. For example, in the single j shell model of ^{44}Ti , the 2 valence protons and 2 valence neutrons are in the $f_{7/2}$ shell. In this model one can form states of isospin $T = 0, 1$, and 2. The $T = 0$ states are isosinglet i.e. they only occur in ^{44}Ti . For $T = 1$ in ^{44}Ti there are analogs in ^{44}Sc and ^{44}V . For $T=2$ the multiplet members, all with $A=44$, are Ca, Sc, Ti, V, and Cr. For non-zero T if one knows the wave function of a state (T, T_z) then acting with the lowering operator T_- one obtains the wave function of the state (T, T_{z-1}) in a neighboring nucleus. The T_+ operator will take us to the state (T, T_{z+1}) .

In this work, which can be regarded as an expansion of previous work by Escuderos and Zamick [1]. We will examine isospin selection rules for isoscalar and isovector magnetic dipole transitions in the single j shell model of ^{48}Cr . Because in the single j-shell model we are at midshell in this nucleus, there are additional selections rules that must be considered.

2 Selection Rules and Calculations for B(M1)'s and B(E2)'s.

Note that in the single j shell ($f_{7/2}$) there are no $J = 1^+$ $T = 0$ in ^{44}Ti . There are such states in the odd-odd nucleus ^{46}V . However, the first even-even nucleus for which there are isospin $T=0$ $J=1^+$ states and isospin $T=1$ $J=0^+$ states in the single j shell configuration is ^{48}Cr . We wish to study such states in this section both in single j and the complete f-p space. We show selected results for B(M1)'s and B(E2)'s from 1^+ states to lower 0^+ and 2^+ states in Tables I and II.

Note that in this single j model space all B(M1)'s from $J = 1^+$ $T = 0$ states to $T = 0$ states vanish for all J_f (0, 1, 2). This can be explained by the fact that in the limited model space ($f_{7/2}$) the isoscalar M1 operator is proportional to $J = (J_p + J_n)$, total angular momentum operator. This operator, acting on a state $|\alpha J m\rangle$ will create a state in the same $|\alpha J\rangle$ multiplet and thus will not induce M1 transitions to different multiplets. Also in this model space all transitions from 1^+ $T = 1$ states to other $T = 1$ states ($J_f = 0, 1, 2$) also vanish. This is a known result which can be related to the vanishing, in an $N = Z$ nucleus, of the Clebsch-Gordan coefficient $(1\ 1\ 0\ 0\ |10)$.

Some of the zeros, however, are specific to ^{48}Cr . In the single j shell we are at midshell. The 4 protons and 4 neutrons can also be regarded as 4 proton holes and 4 neutron holes. As first noted by Escuderos, Zamick and Bayman [2] and shown analytically by Neergaard [3], the quantity $S = (-1)^{(v_p+v_n)/2}$ is a good quantum number, where v_p and v_n are the seniorities of the protons and neutrons respectively. With the MBZE interaction [2] the $J = 0^+$ ground state has $S = +1$. With the same interaction the yrast states of even J have $S = (-1)^{J/2}$ i.e. $S = +1$ for $J = 0_1$, $S = -1$ for $J = 2_1$, and $S = +1$ for $J = 4_1$ etc. Along the yrast chain the B(E2)'s are large and for these we have $S_f = -S_i$. In the single j model space the B(E2)'s for transitions in which $S_f = S_i$ will vanish.

We show a brief example of the selection rules in Table I. We consider B(E2)' from the $J = 0_1^+$ $S=+1$ state first two $J = 2^+$ states. The 2_1^+ state has $S = -1$ whilst the 2_2^+ state has $S = +1$, just like the $J = 0_1^+$ ground state. We use the Shell model Code NUSHELLX of B.A.Brown and W.D.M. Rae[4].

Table I: B(E2)'s in the large and small spaces ($.e^2 \text{ fm}^4$).

0 ₁ To	Large	Single j
2 ₁	1225.5	452.6
2 ₂	2.367	0

With regards to magnetic dipole transitions it is easy to show that for B(M1) not to vanish a necessary condition is that $S_f = S_i$ i.e. $\Delta S = 0$. This can be easily shown by examining the wave functions of MBZE.[2]. When the M1 operator acts on a basis state $[J_p, J_n]$ it creates a state with the same $[J_p, J_n]$ (including any internal quantum numbers). We then overlap with the final state. In the latter only the component with the same $[J_p, J_n]$ will contribute. If $D(J_p, J_n)$ for the initial state is non-zero then the corresponding coefficient for the final state will be non zero only if $S_f = S_i$.

The values of (T,S) for the single j -shell for $J=0^+$ states in ref [2] are respectively ((0,+), (0,-), (0,+), (0,-), (2,+), (0,+), (1,-), (1,-), (0,+), (2,+), (2,+), (2,+).

For $J=1^+$ states the values are (1,+), (1,-), (1,+), (0,-), (1,-), (1,+), (0,+), (0,-), (1,-), (1,+), (1,+), (1,+), (2,-), (1,-), (1,+), (2,-), (3,+).

We next take a casual look at magnetic dipole transitions and look for selection rules for B(M1) values. With the MBZE[2] interaction, the S values for the first 3 $J = 0^+$ $T = 0$ states are +1,-1 and +1 respectively whilst the only 2 $J = 0^+$ $T = 1$ states both have $S = -1$. For $J = 0^+$ $T = 2$ all 3 states have $S = +1$. Hence the first and third $J = 1^+$ $T = 1$ states will connect with all three but the second will not connect with any $J = 0^+$ $T = 2$ states. For the first 3 $J = 1^+$ $T = 0$ states the S values are -1,+1 and -1; for $J = 1^+$ $T = 1$ they are +1,-1, and +1; for $J = 2^+$ $T = 0$ they are -1,+1,+1 and finally, for $J = 2^+$ they are -1,+1,-1.

For the lowest "special" $J = 1^+$ $T = 0$ state which has $S = -1$ there will be no transitions in the single j shell model space to any $T = 0$ states, and there will be non-zero B(M1)'s only to the 2 $J = 0^+$ $T = 1$ states, to the second $J = 1^+$ $T = 1$ state and to the first and third $J = 2^+$ $T = 1$ states.

The above selection rules can be obtained as easy generalizations of results for particles of one kind, as described e.g. in R.D. Lawson's book [4] and based on early work by G. Racah [5]. See especially eq. 3.59 and the discussions that follow. It is there shown that the matrix element of the O^λ operator for particles is related to that for holes by a phase factor $(-1)^{1+\lambda+(v-v')/2}$. In that work v refers to the seniority of particles of one kind. We simply replace v by $(v_p + v_n)$ and v' by $(v'_p + v'_n)$. In order to get a non-vanishing matrix element the phase factor must be positive. For B(M1) $\lambda = 1$ and for B(E2) $\lambda = 2$. This explains the selection rules in a more formal way.

We next consider what happens in the complete f-p shell model space. In Table II we show the large space results from various $J = 1$ $T = 0$ states to lowest and second $J = 0^+$ and $J = 2^+$ states and likewise from various $J = 1^+$ $T = 1$ states. All B(M1)'s in the upper half are $T = 0$ to $T = 0$ transitions and indeed they would have vanished in the single j-shell calculations. In the lower half of Table II we have $T = 1$ to $T = 0$ transitions and indeed the B(M1)'s are on the whole much larger. Let us focus on the lowest $J=1$ $T=1$ to the lowest $J=0$ $T=0$ transition. The value of B(M1) is 1.101. The orbital value is 0.3046 and the spin value is 0.2475. The amplitudes add constructively to give the total B(M1). When considered in reverse i.e.

from 0 to 1 the B(M1) is 3 times as large and is often compared to the idealized purely orbital scissors mode.

Table II: B(M1)'s in a Large Space (μ_N^2)

$J = 1_n^+ T = 0 \rightarrow$	Lowest $J = 0^+$	Second $J = 0^+$	Lowest $J = 2^+$	Second $J = 2^+$
$n = 1$	0.2003 E-3	0.2124 E-4	0.1095 E-4	0.6845E-4
2	0.1343 E-1	0.1334 E-3	0.5432 E-2	0.1288 E-3
E-3	0.5903 E-3	0.9063 E-5	0.5268 E-4	0.7698 E-4
4	0.5461 E-4	0.2338 E-2	0.4631 E-4	0.3361 E-2
5	0.7451E-5	0.1677 E-5	0.2297 E-3	0.1098 E-2
$J = 1_n^+ T = 1 \rightarrow$				
$n = 1$	0.1101 E+1	0.1221 E-1	0.4665 E0	0.5316 E-1
2	0.6551 E0	0.1813 E0	0.3838 E0	0.4569 E-1
3	0.1570 E0	0.2142 E0	0.9775E-1	0.6353 E0
4	0.2353 E0	0.1709 E0	0.1768 E-2	0.4243 E0
5	0.5526 E-1	0.2574 E0	0.4835 E-2	0.2511 E0

In Table III we show B(E2)'s from various $J = 1^+ T = 0$ states to the 2 lowest $J = 2^+$ and likewise from various $J = 1^+ T = 1$ states. The largest B(E2) in Table III is 25.32 e² fm⁴. This is considerably smaller than value 1225.5 e² fm⁴ for the collective $J = 0^+ \rightarrow J = 2^+$ transition shown in Table I.

Table III: B(E2)'s in a Large Space(e²fm⁴) .

$J = 1_n^+ T = 0 \rightarrow$	Lowest $J = 2^+$ State	Second $J = 2^+$ State
$n = 1$	0.1897 E2	0.4232 E-2
2	0.5243 E1	0.5807 E0
3	0.8518 E-1	0.2532 E2
4	0.2944 E0	0.1551 E-1
5	0.8348 E-3	0.4874 E-1
$J = 1_n^+ T = 1 \rightarrow$		
$n = 1$	0.6694 E1	0.1909 E-1
2	0.4376 E1	0.2359 E-1
3	0.3735 E0	0.2125 E1
4	0.1676 E-2	0.7543 E0
5	0.1546 E0	0.5553E-1

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3 Appendix

. As an example we show the wave functions of the first 2 $J=0^+ T=0$ states in ⁴⁸Cr from Ref [2].
 .Table IV Selected single j-shell wave functions with the MBZE interaction[2] in ⁴⁸Cr.

	J=0 ⁺	J=0 ⁺
Energy (MwV)	0.0000	5.5785
$J_P J_n$		
00	0.7494	0
22	0.5445	0
22*	0	0.6738
2*2	0	0.6738
2*2*	0.1243	0
44	0.1951	0
44*	0	0.2144
4*4	0	0.2144
4*4*	0.2521	0
5*5	0.0932	0
66	0.1231	0
8*8*	0.0393	0

. In Table IV the * designates a seniority 4 basis state. The wave functions of MBZE are written as follows:

$$\Psi = \sum D^{\alpha J}(J_p, J_n)[J_p, J_n]^J \quad (1)$$

The value e.g. 0.5445 is the probability amplitude that in the lowest J=0⁺ state the 2 protons couple to angular momentum 2 and likewise the 2 neutrons. The lowest J=0⁺ state has S=+1 and the next one has S=-1. From ref [2] we see that there are 4 J=0⁺ T=0 S=+1 states and 2 J=0⁺ T=0 S=-1 states. This is true for any charge independent interaction. For J=1⁺ T=0 all states have S=-1.

References

- [1] A. Escuderos and L.Zamick ,Romanian Journal of Physics, Vol. 58, Nos. 9-10, pp 1064-1075, (2013)
- [2] A.Escuderos, L.Zamick and B.F. Bayman, arXiv:nucl-th/0506050 (2006)
- [3] K. Neergaard , Phys. Rev. C 91, 044313 (2015)
- [4] The Shell- Model Code NUSHELLX@MSU , B.A. Brown and W.D.M. Rae ,
<http://www.sciencedirect.com/science/article/pii/S0090375214004748>
- [5] R.D. Lawson, Theory Of The Nuclear Shell Model, Clarendon Press, Oxford (19080)
- [6] G. Racah, Phys. Rev. 63, 367 (1943).